

Linear Optimization Models with Integer Solutions for Ping Control Problems in Multistatic Active Acoustic Networks

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Abstract—The typical ping control objective for a multistatic network is to optimize a sonar performance metric within a search area over the mission scenario time horizon subject to available ping energy at sources. Assuming sonar performance models and near real-time multistatic target trackers are available, sonar performance metric predictions for near-future time window can be obtained. However, for Anti-Submarine Warfare applications, it is not realistic to accurately predict performance metrics involving an unknown number of evasive targets over a long scenario time horizon. Therefore, effective and efficient ping control methods must consider effective strategies to obtain desired performance over the spectrum of operational objectives and time frames.

In this paper, we develop four integer-linear goal programming models to provide intelligent ping control decisions for various operational modes that depend on remaining ping energy and remaining scenario time. We incorporate the multiple objectives of: maximizing the sonar performance metric, judicious use of energy-limited sources and maintaining a certain level of ping activity. We show that the constraint matrix of each relaxed linear model possesses the total unimodularity property, guaranteeing optimal integer ping control solutions. Therefore, computationally efficient linear programming methods can be used in the implementation of these models. We simulate multiple operational scenarios and demonstrate the properties of the resulting ping strategies in terms of the performance metric and individual source and network lifetime. Results are compared to the baseline ping strategy, which considers the sonar performance metric alone.

Keywords—Ping control, sensor management, ping management, sonar optimization, multistatic tracking.

I. INTRODUCTION

Multistatic active acoustic networks provide Anti-Submarine Warfare (ASW) capability to detect, localize, and track threat targets through the expanded geometric diversity of a distributed field of sources and receivers [1]. However, given the variabilities in acoustic environmental conditions, sensor performance and threat target behavior, such networks cannot exploit their full potential without management and control methods. Control methods may be applied to the ping scheduling task to obtain improved detection, localization and tracking performance.

The proper implementation of a ping control model within a multistatic network will depend on the current mission “mode of operation”. In the “Target Search” mode of operation, the objective is to quickly detect any targets present within the surveillance area and initiate tracking using these detections. If detections are not obtained (when performance predictions indicate targets should be detectable), there is more confidence that the area is effectively cleared or sanitized of target threats. In the “Track-Holding” mode of operation, the objective is to maintain high-quality track estimates for those targets that have already been detected. In the “Search and Hold” mode of operation, the objective is to perform the previous two modes in parallel: maintain tracking on detected targets while continuing to search for undetected targets. It may be that even within the operational scenario, the mission mode may change from one to another. This paper describes a dynamic ping control decision methodology that is applicable over this wide range of mission modes and operational scenarios, while also considering energy constraints of the system’s acoustic sources.

Ping control algorithms require a metric for the predicted sonar performance obtained by pinging a given source at a particular time. A number of performance metrics may be selected for this purpose. Among them are: average detection probability during search mode with confirmed detections [2], probability of target presence during search mode without confirmed detections [3], probability of target presence during search and/or track-hold mode with confirmed detections [4], and ASW residual risk metric for area clearance [5]. Predictions are of most value to the control process within a near-future time window. Given the prohibitive task of accurately and efficiently forecasting sonar performance metrics for a far-future time window, in our case we assume the probability of target presence is uniform over the search area and use a constant, nominal sonar performance metric for each source as given by a sonar performance model. Although the accuracy of predicted performance metrics is important, in this paper we focus on obtaining the ping decision assuming appropriate sonar performance metrics are given.

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Previously published work on ping control methods for multistatic active acoustic networks include [2], [6], and [7]. In [6], a greedy approach is used to select a ping source which optimizes one-step prediction of the expected detection performance metric during both search and track-hold modes. In [7], another greedy approach is presented which selects not only a ping source but also a waveform type and fine-tuned ping timing adjustments to obtain high-strength specular echoes. It also optimizes the one-step prediction of the detection performance metric, but only for the track-hold mode. However, the greedy approaches using a one-step performance metric prediction do not consider the remaining ping energy at each source. This may result in over-usage of certain sources (risking too early expenditure to complete the mission) or ping waste (pinging when no/little benefit is expected). In [2], an approximate dynamic programming approach with sampling-based policy rollout implementation is used to address the energy constraint at sources (during search mode) with the goal of extending network lifetime. The approach is to obtain an optimized ping sequence solution (including quiescent periods, as needed) over the entire scenario time horizon. Such an approach may extend the network lifetime but optimizing based only on the detection performance metric may result in early expiration of certain sources. Further, policy rollout implementation to generate performance metric predictions over a long time horizon is computationally expensive and the suitable prediction accuracy for future performance metrics is hard to achieve due to unknown target behaviors.

In this paper we develop four integer-linear goal programming models to address the wide spectrum of operational modes previously mentioned. Each of these will depend on the remaining time in the operational scenario and the remaining ping energy at each source. We assume a capability exists to estimate performance metrics for each source within the near-future time frame and a constant, nominal performance metric is used for each source within the far-future time frame. The optimization models will consider the following multiple objectives: maximizing the performance metric, balancing ping source usage, and maintaining a certain level of ping activity.

Further, we show that these integer-linear models can be solved with highly efficient linear programming methods and that the relaxed linear models are guaranteed to provide integer optimal solutions. We simulate multiple operational scenarios and demonstrate the properties of the resulting ping strategies in terms of the sonar performance metric and individual source and network lifetime. Results are compared to the baseline ping strategy, which considers the sonar performance metric alone. This paper proceeds as follows. In Section II, we describe the goal programming approach as an optimization modeling method for various operational modes. We also discuss the ping control framework within which such optimization models would operate. In Section III, we describe the detailed formulations of four integer-linear goal programming models. In Section IV, we prove the integer solution property of each linear relaxation model, which allows computationally efficient implementation. Section V presents simulation results

that demonstrate the properties of ping solutions generated by each of the optimization models. Conclusions and future work are provided in Section VI.

II. OPTIMIZATION MODELING APPROACH

In previous work on ping control optimization, it has been assumed that ping optimization can be encompassed within a single overriding objective, such as maximizing detection probability. However, this assumption is not realistic, especially in the ASW applications where forecasting detection performance metrics over the scenario time horizon may be prohibitive. Ping control optimization should also focus on other operational objectives, such as maintaining area coverage capability over the scenario time horizon, extending network lifetime, preventing long ping gaps in addition to maximizing the detection metric. The goal programming approach provides a way of striving toward multiple objectives simultaneously [8]. We develop four goal programming models, each one corresponding to different operational modes and energy supply levels. In the following, we first provide a brief description of a general closed-loop ping control framework in which the optimization models serve as main components.

The ping control framework can be summarized as follows: given the scenario (mission mode, remaining scenario time, and remaining energy) and the assumed performance metrics, a user or automated rule-set determines which ping optimization model is to be solved. It provides a solution for the ping source sequence for discretized ping times over the remaining scenario time. The first k (less than or equal to the scenario time window) ping sources are selected and pings are generated accordingly. Detections from the k pings are processed by a multistatic target tracker and the performance metric predictions for the remaining time window are then updated. At this point the optimization procedure iterates. An optimization model is then selected based on the updated scenario and solved to generate a new ping sequence. This process is repeated until the end of the operational scenario time window. The faster the performance metric update rate (i.e. smaller k), the more accurate the performance metric predictions can be made, and thus a more effective and efficient ping solution can be achieved.

We focus our discussion on four optimization models to be used within the framework. Fig. 1 shows the four optimization models. The Baseline model (P1) is activated for tracking particular target(s) over a relatively short time window with the overriding objective of maximizing performance to maintain continuous track-hold on targets already detected. Unlimited or abundant energy is assumed to be available in this case. The other models, (P2), (P3), and (P4), may be activated for target search and/or search-hold modes.

The Energy Reservation model (P2) is activated when the ping energy is limited at certain sources but enough total ping energy is available to cover the scenario time window. The optimization objectives for (P2) are maximizing the performance metric over the scenario time window and preventing the over usage of the sources with limited energy. This prevents

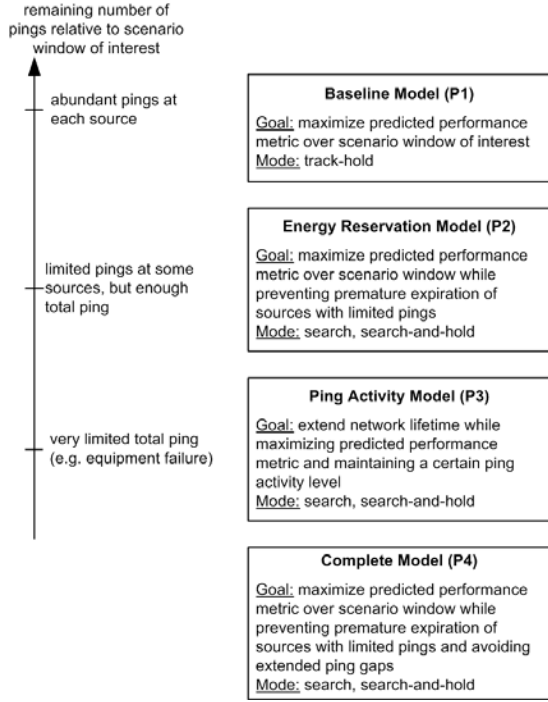


Figure 1. Goal programming optimization models that depend on remaining ping energy and operational scenarios.

premature expiration of those sources, thereby maintaining area coverage over the scenario time window. To include the latter objective, we divide the total scenario window into subintervals and allocate ping energy from each source to each subinterval. A penalty is applied when there is over usage of ping energy by sources that have limited energy. The allocation of a source's remaining ping energy is distributed approximately uniformly over the subintervals.

The Ping Activity model (P3) is activated when the total ping energy is very limited with respect to planned scenario time window. An example of this is the case of equipment failure, and the additional optimization objective is to extend the network lifetime while maintaining a certain minimum ping activity level (i.e., no long ping gaps) until all ping energy is exhausted. To prevent long gaps between successive pings, we introduce an m -of- n ping activity rule, which is to generate at least m pings over the n discretized ping times. The penalty for violating the m -of- n rule is then included in the total objective function of (P3).

The Complete model (P4) may be activated when the ping energy is limited at certain sources as in (P2), or when there is not enough total ping energy for the planned operating time window as in (P3). Here, the goal is to strategically distribute null pings over the time window to last the duration of the scenario, without significantly sacrificing detection performance. In this model, both the energy reservation objective of (P2) and the ping activity objective of (P3) are included, in addition to the sonar performance objective.

III. FOUR INTEGER-LINEAR OPTIMIZATION MODELS

In this section, we provide mathematical descriptions for four integer-linear goal programming models, which incorporate the additional operational goals described in Section II. We start by listing the notation that is used in the rest of the paper. We denote

Index sets and parameters:

N_S = number of sources,

$J = \{1, 2, \dots, N_S\}$,

T = number of discretized ping times in the remaining scenario window,

$P = \{1, 2, \dots, T\}$,

N_Δ = number of subintervals in the remaining scenario window,

$Q = \{1, 2, \dots, N_\Delta\}$,

Δ_q = set of consecutive ping times in P in subinterval q , $q \in Q$,

e^j = remaining number of pings at source j , $j \in J$,

Δe_q^j = allocated number ping energy for source j in subinterval q , $j \in J, q \in Q$,

m = lower bound of number of pings in m -of- n ping activity rule,

n = number of consecutive ping times in m -of- n ping activity rule,

N_δ = number of running time intervals over the remaining scenario time window,

$R = \{1, 2, \dots, N_\delta\}$,

δ_r = set of consecutive ping times in P in running time interval r , $r \in R$,

Performance parameters:

c_p^j = performance metric of source j at ping time p , $j \in J, p \in P$,

a_q^j = penalty coefficient for over utilization of source j in subinterval q , $j \in J, q \in Q$,

b_r = penalty coefficient for under pinging in running time interval r , $r \in R$,

Decision variables:

x_p^j = binary ping decision variable for source j at ping time p , $j \in J, p \in P$,

x_p^j is 1 if allocated, 0 otherwise,

y_q^j = over utilization integer decision variable for source j in subinterval q , $j \in J, q \in Q$,

z_r = under pinging integer decision variable in running time interval r , $r \in R$.

We illustrate the notations of time intervals with a small example in Fig. 2. In this example, the scenario time window is discretized into 12 ping times ($T = 12$). The scenario time window is divided into two subintervals ($N_\Delta = 2$), with Δ_1 consisting of ping times $\{1, 2, \dots, 7\}$ and Δ_2 of ping times $\{8, 9, \dots, 12\}$. We define a running time interval of length 4 ($n = 4$, $N_\delta = T - n + 1 = 9$), with δ_r consisting of ping times $\{r, r + 1, r + 2, r + 3\}$ for $r = 1, 2, \dots, N_\delta$.

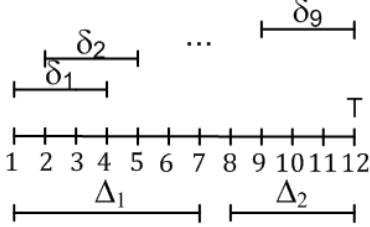


Figure 2. Discretization of remaining scenario time window.

The complete problem formulation (P4) is given by:

$$\max \sum_{p \in P} \sum_{j \in J} c_p^j x_p^j - \sum_{q \in Q} \sum_{j \in J} a_q^j y_q^j - \sum_{r \in R} b_r z_r \quad (1)$$

$$\text{sub. to } \sum_{p \in P} x_p^j \leq e^j, \text{ for all } j \in J, \quad (2)$$

$$\sum_{j \in J} x_p^j \leq 1, \text{ for all } p \in P, \quad (3)$$

$$\sum_{p \in \Delta_q} x_p^j - y_q^j \leq \Delta e_q^j, \text{ for all } j \in J, q \in Q, \quad (4)$$

$$\sum_{p \in \delta_r} \sum_{j \in J} x_p^j + z_r \geq m, \text{ for all } r \in R, \quad (5)$$

$$x_p^j \text{ is binary, for all } j \in J, p \in P, \quad (6)$$

$$y_q^j \text{ is nonnegative integer, for all } j \in J, q \in Q, \quad (7)$$

$$z_r \text{ is nonnegative integer, for all } r \in R. \quad (8)$$

The objective function (1) includes the total sonar performance metric summing responses over all ping sources and all ping times in the scenario time window; the total penalty metrics summing over-utilization of the allotted pings over all sources and all subintervals; and the total penalty metrics summing under-pinging according to the m -of- n activity rule over all running intervals. The set of constraints include:

- Constraint (2): For each source, total ping usage over the scenario window must not exceed available energy.
- Constraint (3): At each ping time, at most one source is selected.
- Constraint (4): For each source and for each subinterval, the number of allocated pings over the available pings is accounted for, so that each unit of over-usage can be penalized in the objective function.
- Constraint (5): For each running time interval, the number of allocated pings below a desired number of pings is accounted for, so that each unit of under-usage can be penalized in the objective function.

- Constraints (6) – (8): Decision variables must be integers.

The four optimization models correspond to four subproblems of the complete problem; the inclusion of certain terms in the objective and constraint functions depends on the mission mode and the remaining pings as described in Section II. The four subproblems are:

- Baseline model (P1): The objective function includes the first term of (1), and the constraint functions are given by (2), (3), and (6).
- Energy reservation model (P2): The objective function includes the first two terms of (1), and the constraint functions are given by (2), (3), (4), (6) and (7).
- Ping activity model (P3): The objective function includes the first and last terms of (1), and the constraint functions are given by (2), (3), (5), (6) and (8).
- Complete model (P4): It is equivalent to the complete problem (1) – (8).

The optimization problems given above include the constraints that each decision variable (x_p^j, y_q^j, z_r) must be integer. We will consider a linear programming relaxation to these problems where the integrality constraints are dropped. The resulting linear programs can be solved efficiently by the simplex method or other polynomial complexity methods. In the next section, we show that optimal solutions to the relaxed problems are guaranteed to be integer solutions, and therefore are also optimal solutions to the corresponding integer-constrained problems.

IV. INTEGER SOLUTION PROPERTY

We show each of the four integer-linear models described in Section III can be solved as a relaxed linear model and the relaxed model satisfies the integer solution property. We apply the following well-known result from Operations Research that provides the conditions for integer solutions even if integer constraints are relaxed.

Theorem (Hoffman-Kruskal Theorem) (cf. [10]): *Let A be an m by n integral matrix. Then the polyhedron defined by $Ax \leq b, x \geq 0$ is integral for every integral vector $b \in \mathbf{R}^m$ if and only if A is totally unimodular.*

Proof: See [10], pp. 221. ■

Definition: A matrix is defined as “totally unimodular” if all of its square submatrices have determinant 0, 1, or -1 .

Three specially structured matrices with the totally unimodular (TU) property are of interest. These special matrices are:

- node-edge incidence matrix of a bipartite undirected network,
- node-edge incidence matrix of a directed network,
- binary matrix with consecutive-ones property, i.e., in each row the 1's appear consecutively (This matrix is also referred to as an interval matrix).

The TU property for these special matrices is described in [11] and [12].

A. Baseline Model (P1)

We consider a linear optimization model with constraints (2) and (3). For ease of presentation, we consider the following

example with 2 sources ($N_S = 2$) and 5 ping times ($T = 5$). The corresponding constraint matrix, A_1 , of the equivalent linear model is given as:

$$A_1 = \begin{bmatrix} A \\ B \end{bmatrix},$$

where

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

In general, A and B have the dimensions of $N_S \times (N_S \times T)$ and $T \times (N_S \times T)$ respectively.

We see that A_1 represents a node-edge incidence matrix of a bipartite undirected network (Fig. 3) and thus A_1 is totally unimodular [11]. In Fig. 3, an edge (s, p) corresponds to a column in A_1 which has exactly two 1s; one 1 in each of the rows corresponding to nodes s and p .

The column vector b_1 resulting from the column-wise concatenation of the right-hand-sides of (2) and (3) is integral. Therefore, from the Hoffman-Kruskal Theorem, the baseline model (P1) satisfies the integer solution property.

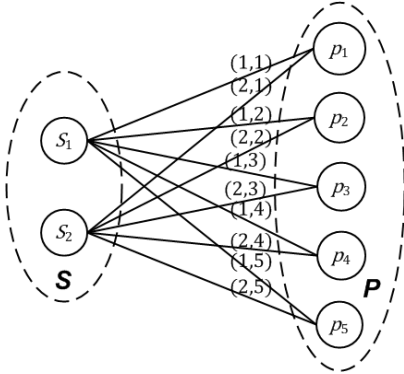


Figure 3. Bipartite network representing the constraint matrix A_1 for an example with two sources $\{s_1, s_2\}$ and five ping times $\{p_1, p_2, p_3, p_4, p_5\}$. Edge (i, j) represents an edge incident to nodes s_i and p_j .

B. Energy Reservation Model (P2)

Now consider the linear optimization model with constraints (2), (3) and (4). We show that the constraint matrix of (P2) is equivalent to a network matrix. We first replace the inequality constraints (2), (3) and (4) by a system of equality constraints:

$$\sum_{p \in P} x_p^j + u^j = e^j, \text{ for all } j \in J, \quad (9)$$

$$\sum_{j \in J} x_p^j + v_p = 1, \text{ for all } p \in P, \quad (10)$$

$$\sum_{p \in \Delta_a} x_p^j - y_p^j + w_q^j = \Delta e_q^j, \text{ for all } j \in J, p \in P, \quad (11)$$

where u^j , v_p and w_q^j are non-negative slack variables. In order to view the resulting constraint matrix as a balanced network matrix, we include two additional dummy constraints without affecting the solution to the original optimization model:

$$\sum_{j \in J} u^j = \max\{0, T - \sum_{j \in J} e^j\}, \quad (12)$$

$$\sum_{p \in P} v_p = \max\{0, \sum_{j \in J} e^j - T\}. \quad (13)$$

Again, we utilize the example with 2 sources ($N_S = 2$) and 5 ping times ($T = 5$) for the ease of presentation. In this example, we subdivide the scenario window into 2 subintervals ($N_\Delta = 2$) with Δ_1 consisting of ping times $\{1, 2, 3\}$ and Δ_2 of ping times $\{4, 5\}$. The corresponding constraint matrix, A_2 , for the new equivalent linear model is given by:

$$A_2 = \begin{bmatrix} A & 0 & I_2 & 0 & 0 \\ B & 0 & 0 & I_5 & 0 \\ C & -I_4 & 0 & 0 & I_4 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & b & 0 \end{bmatrix},$$

where

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix},$$

A and B are as previously defined, a and b are row vectors of 1s of size 2 and 5 respectively, the 0s in A_2 are zero matrices of appropriate dimensions, and I_n corresponds to an identity matrix of size n . In general, C , a and b have the dimensions of $(N_S \times N_\Delta) \times (N_S \times T)$, $1 \times N_S$, and $1 \times T$ respectively.

We see that the rows of A and C are linearly dependent. Using elementary row operations, we can rewrite A_2 in an alternative equivalent form, A'_2 as:

$$A'_2 = \begin{bmatrix} 0 & D & I_2 & 0 & -D \\ -B & 0 & 0 & -I_5 & 0 \\ C & -I_4 & 0 & 0 & I_4 \\ 0 & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & b & 0 \end{bmatrix},$$

where

$$D = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Each column of A'_2 has exactly one 1 and exactly one -1 , and the rest of the entries are zeros. We recognize that A'_2 is a node-edge incidence matrix of a directed network (Fig. 4) and represents the constraint matrix of the minimum cost flow problem, which is totally unimodular [11].

The column vector b_2 resulting from the column-wise concatenation of the right-hand-sides of (9) – (13) is integral.

The elementary row operations result in an integral column vector b'_2 . Therefore, from the Hoffman-Kruskal Theorem, the energy reservation model (P2) satisfies the integer solution property.

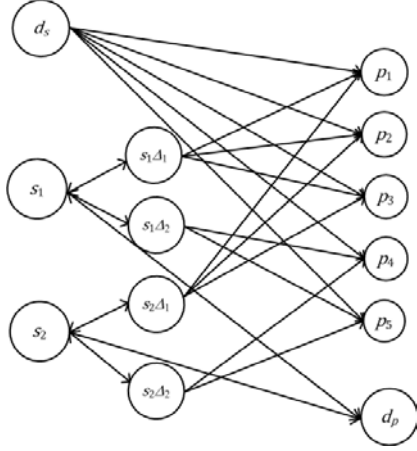


Figure 4. Network representing the constraint matrix A'_2 . In this example, there are three sources (including a dummy source d_s) $\{s_1, s_2, d_s\}$, six ping times (including a dummy ping time d_p) $\{p_1, p_2, p_3, p_4, p_5, d_p\}$ and four source-subinterval pairs $\{s_1\Delta_1, s_1\Delta_2, s_2\Delta_1, s_2\Delta_2\}$.

We have shown that constraint matrices of (P1) and (P2) can be represented as networks, and therefore (P1) and (P2) can be solved very efficiently using a specialized network algorithm, such as the network simplex method.

C. Ping Activity Model (P3)

We consider the linear optimization model with constraints (2), (3) and (5). Again, we use the example with 2 sources ($N_S = 2$) and 5 ping times ($T = 5$) and set $n = 3$ ($N_\delta = T - n + 1 = 3$). The constraint matrix, A_3 , is given by

$$A_3 = \begin{bmatrix} A & 0 \\ B & 0 \\ -E & -I_3 \end{bmatrix},$$

where

$$E = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

and A and B are as defined previously. In general, E has the dimensions of $N_\delta \times (N_S \times T)$.

To show the TU property of the constraint matrix, we first convert the inequality constraints into equality constraints using slack variables. The resulting constraint matrix, A'_3 , is then transformed into an alternative equivalent form using elementary row operations, which do not change the solution of the linear system. Adding slack variables, equality constraint matrix, A'_3 , has the following form:

$$A'_3 = \begin{bmatrix} A & 0 & I_2 & 0 & 0 \\ B & 0 & 0 & I_5 & 0 \\ -E & -I_3 & 0 & 0 & I_3 \end{bmatrix}.$$

The pattern of E , which consists of rows of consecutive ones representing the running time intervals, is different from the network pattern shown in (P1) and (P2).

We perform elementary row operations on A'_3 . The rows of E and B are linearly dependent and the resulting transformed matrix A''_3 is given by:

$$A''_3 = \begin{bmatrix} A & 0 & I_2 & 0 & 0 \\ B & 0 & 0 & I_5 & 0 \\ 0 & -I_3 & 0 & F & I_3 \end{bmatrix},$$

where

$$F = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

The consecutive-ones property of E is preserved in F , and therefore F is totally unimodular [12]. Since $[A \ B]^T$ and F are totally unimodular, we can show that A''_3 is totally unimodular. We first show that the following matrix

$$G = \begin{bmatrix} A & 0 \\ B & I \\ 0 & F \end{bmatrix}$$

is totally unimodular. The proof is included in the appendix in the form of a lemma. Since G is TU, it can be easily shown using a similar argument in the lemma that the matrix resulting from concatenating G row-wise by $[0 \ 0 \ I]^T$, $[0 \ 0 \ -I]^T$ or $[I \ 0 \ 0]^T$ is still TU. Therefore, it is concluded that A''_3 is totally unimodular.

The column vector b''_3 resulting from the column-wise concatenation of the right-hand-sides of equality constraints is integral. Therefore, from the Hoffman-Kruskal Theorem, the ping activity model (P3) satisfies the integer solution property.

D. Complete Model (P4)

Using similar reasoning as in (P3), it can be shown that (P4) satisfies the integer solution property.

V. SIMULATION RESULTS

In this section, we demonstrate the properties of ping control solutions of optimization models (P1) to (P4). First, we need sonar performance metric predictions c_p^j for source $j \in J$ and ping time $p \in P$. In the following examples, we specify four sources ($N_S = 4$) and a remaining scenario time window of length 60 ($T = 60$). We generate random numbers between 0 and 1 to represent the detection performance metric predictions for the four sources for each of the first five ping times. Constant random numbers are specified for the remaining ping times between 6 and 60 to represent the detection performance metrics of four sources with uniform target distribution in the search area. Fig. 5 shows simulated performance metric predictions for four sources over the scenario time window.

Depending on the specified remaining pings for each source and other user-input parameters (criteria for energy limited or very limited scenario, criteria for Δ_q and δ_r , penalty costs, etc.), the algorithm then transforms the problem into one of the linear programming models $\{(P1), \dots, (P4)\}$ and uses

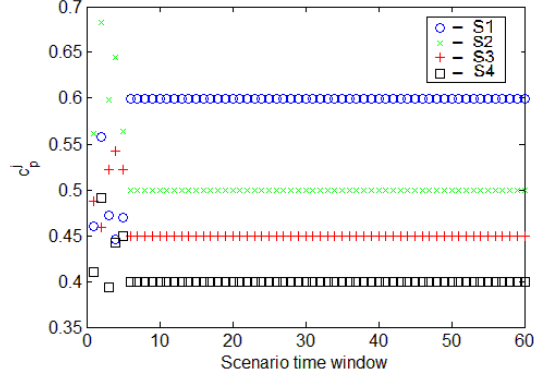


Figure 5. Simulated detection performance metric predictions for four sources over the scenario time window.

lp_solve [13] to obtain the ping control solutions. We generate three examples, examples (a), (b) and (c), corresponding to operational conditions for (P2), (P3), and (P4). We also solve (P1) for each of the operational conditions and use its solution as a baseline comparison to the solution of other models.

Example (a) represents an operational condition for (P2) with limited pings at some sources but enough total number of pings for the scenario window. The remaining pings at sources 1 to 4 ($e^j, j = 1, 2, 3, 4$) are 20, 7, 23, 10 respectively. The scenario time window of 60 ping times ($T = 60$) is divided into 5 subintervals ($N_\Delta = 5$) with each subinterval consisting of 12 ping times. The penalty metric for over utilization, a_q^j , is set to 0.3 for sources 1 and 3 and 0.8 for sources 2 and 4 for each subinterval $\Delta_q, q = 1, 2, \dots, 5$. The models (P1) and (P2) are solved and the ping control results are shown in Fig. 6. For the first 5 ping times, the sum of the detection performance metrics of the (P1) solution is greater than that of the (P2) solution by 0.15. For the whole scenario window of length 60, the difference in the sums of detection performance metrics is only 0.05. Considering detection performance metric alone, the (P1) ping control strategy exhausts ping energy of sources 1 and 2 before the middle of the planned scenario window. With the inclusion of energy reservation objective, the (P2) ping control strategy reserves some of the ping energy of sources 1 and 2 for usage near the end of the scenario window.

Example (b) represents an operational condition for (P3) with very limited total number of pings. The remaining pings at sources 1 to 4 are 10, 1, 7, 5 respectively. The m -of- n ping activity rule is set to 3-of-5. The penalty metric for under pinging in each running interval, b_r , is set to $0.5\gamma^{r-1}$, where γ is a discount factor of 0.9 and $r \in R = \{1, 2, \dots, 56\}$. The models (P1) and (P3) are then solved and the ping control results are shown in Fig. 7. The sum of the detection performance metrics of the (P1) solution is greater than that of the (P2) solution by only 0.04 over the network lifetimes. However, the network lifetime of the (P3) ping control strategy exceeds that of the (P1) strategy by 15 ping times.

Example (c) represents an operational condition for (P4)

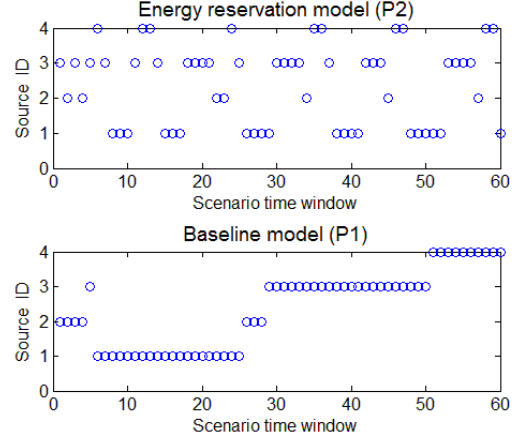


Figure 6. Example (a) – Ping decision comparison between baseline model (P1) and energy reservation model (P2), where a circle indicates a ping. A circle at source ID 0 (null ping) represents no ping.

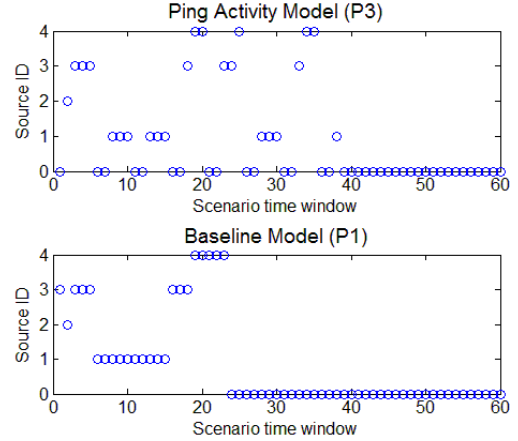


Figure 7. Example (b) – Ping decision comparison between baseline model (P1) and ping activity model (P3), where a circle indicates a ping. A circle at source ID 0 (null ping) represents no ping.

with the total number of pings available is less than the scenario time window of 60 but not as limited as in an operational condition for (P3). The remaining pings at sources 1 to 4 are 7, 15, 12, 10 respectively. The parameters for energy reservation and ping activity objectives are the same as in examples (a) and (b). The models (P1) and (P4) are then solved and the ping control results are shown in Fig. 8. The sum of the detection performance metrics of the (P1) solution is greater than that of the (P2) solution by only 0.02 over the network lifetimes. However, the network lifetime of the (P4) ping control strategy exceeds that of the (P1) by 14 ping times. In addition, the (P4) ping control strategy reserves some ping energy of all available sources for usage in all subintervals, while the (P1) strategy exhausts sources 1 and 2 energy before the middle of the planned scenario time window.

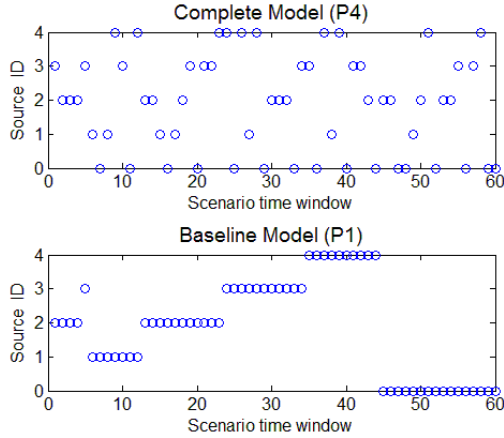


Figure 8. Example (c) – Ping decision comparison between baseline model (P1) and complete model (P4), where a circle indicates a ping. A circle at source ID 0 (null ping) represents no ping.

VI. CONCLUSIONS AND FUTURE WORK

We have presented four integer-linear goal programming optimization models to address ping control problems in multistatic sonar networks for various ASW operational scenarios. We have introduced two additional optimization objectives: ping energy reservation and maintaining a certain ping activity level. These are used together with the typically utilized objective of maximizing a suitable sonar performance metric to provide robust ping control solutions. Each of the optimization models can be solved using highly efficient linear programming methods, due to the fact that the solutions to these models are guaranteed to produce integer solutions, which has been proved. The efficiency and effectiveness of the models are illustrated with simulated data representing various scenarios. Future work will include the integration of the optimization models with an actual sonar performance model and multistatic target tracker. This will provide a realistic simulation capability with which comprehensive evaluation of the ping control methods for ASW scenarios may be made.

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APPENDIX

Lemma: *The matrix*

$$G = \begin{bmatrix} A & 0 \\ B & I \\ 0 & F \end{bmatrix},$$

has the totally unimodular property, given that $[A \ B]^T$ and F are totally unimodular.

Proof: Let \tilde{G} be an $n \times n$ submatrix of G constructed from submatrices of A, B, I and F which are denoted by $\tilde{A}, \tilde{B}, \tilde{I}$ and \tilde{F} respectively, i.e.

$$\tilde{G} = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{B} & \tilde{I} \\ 0 & \tilde{F} \end{bmatrix}.$$

Let the dimensions of \tilde{A} and \tilde{F} be $p \times r$ and $q \times (n - r)$, where $p, q, r < n$ respectively. The dimensions of \tilde{B} and \tilde{I} follow accordingly. If $p = r$ (\tilde{A} is square), then $[\tilde{I} \ \tilde{F}]^T$ is square and $\det(\tilde{G}) = \det(\tilde{A}) \cdot \det([\tilde{I} \ \tilde{F}]^T)$ equals $-1, 0$, or 1 . Similarly, if $q = n - r$ (\tilde{F} is square), then $[\tilde{A} \ \tilde{B}]^T$ is square and $\det(\tilde{G}) = \det([\tilde{A} \ \tilde{B}]^T) \cdot \det(\tilde{F})$ equals $-1, 0$, or 1 .

If $p > r$, then $q < (n - r)$, and \tilde{G} can be rewritten as:

$$\tilde{G} = \begin{bmatrix} [\tilde{A} \ 0] & 0 \\ [\tilde{B} \ \tilde{I}_1] & [\tilde{I}_2] \\ [0 \ \tilde{F}_1] & [\tilde{F}_2] \end{bmatrix}.$$

The submatrices of \tilde{G} are regrouped such that $[\tilde{A} \ 0]$ and $[\tilde{I}_2 \ \tilde{F}_2]^T$ are square matrices of dimensions $p \times p$ and $(n - p) \times (n - p)$ respectively. It follows that $\tilde{I}_1, \tilde{I}_2, \tilde{F}_1, \tilde{F}_2$ are submatrices of \tilde{I} and \tilde{F} with appropriate dimensions. Then, $\det(\tilde{G}) = \det([\tilde{A} \ 0]) \cdot \det([\tilde{I}_2 \ \tilde{F}_2]^T) = 0$.

If $q > (n - r)$, then $p < r$, and \tilde{G} can be rewritten as:

$$\tilde{G} = \begin{bmatrix} [\tilde{A}_1] & [\tilde{A}_2 \ 0] \\ [\tilde{B}_1] & [\tilde{B}_2 \ \tilde{I}] \\ 0 & [\tilde{F}] \end{bmatrix}.$$

The submatrices of \tilde{G} are regrouped such that $[\tilde{A}_1 \ \tilde{B}_1]^T$ and $[0 \ \tilde{F}]$ are square matrices of dimensions $(n - q) \times (n - q)$ and $q \times q$ respectively. It follows that $\tilde{A}_1, \tilde{A}_2, \tilde{B}_1, \tilde{B}_2$ are submatrices of \tilde{A} and \tilde{B} with appropriate dimensions. Then, $\det(\tilde{G}) = \det([\tilde{A}_1 \ \tilde{B}_1]^T) \cdot \det([0 \ \tilde{F}]) = 0$. Therefore G is totally unimodular. ■